

Status Update

Neutrino Oscillation In Matter

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Outlines

- Neutrino oscillation in vacuum
 - Survival probability
 - Transition probability
- Neutrino oscillation in matter
 - Effective potential in matter
 - MSW effect
 - General form of oscillation probability in matter
 - Survival probability
 - Transition probability
- Next 2 weeks plan

General form of oscillation probability in matter

- By calculating the perturbation to the first order in a and Δm_{21}^2 . We obtain

$$\begin{aligned} (S(x))_{\beta\alpha} &= (S_0(x))_{\beta\alpha} + (S_1(x))_{\beta\alpha} \\ &= \delta_{\alpha\beta} - i2e^{-i\frac{\Delta m_{31}^2 x}{4E}} \sin \frac{\Delta m_{31}^2 x}{4E} U_{\beta 3} U_{\alpha 3}^* \times \\ &\quad \left[1 - \frac{a}{\Delta m_{31}^2} (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}) - \frac{iax}{2E} |U_{13}|^2 \right] \\ &\quad - i\frac{\Delta m_{31}^2 x}{2E} \times \\ &\quad \left[\frac{\Delta m_{21}^2}{\Delta m_{31}^2} U_{\beta 2} U_{\alpha 2}^* + \frac{a}{\Delta m_{31}^2} (\delta_{\alpha 1} \delta_{\beta 1} + U_{\beta 3} U_{\alpha 3}^* (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1})) \right] \end{aligned} \quad (1)$$

Neutrino oscillation in matter

Then the oscillation probability is

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= |S_{\beta\alpha}(L)|^2 \\ &= \delta_{\alpha\beta} \left[1 - 4|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \left(1 - \frac{2a}{\Delta m_{31}^2} (|U_{13}|^2 - \delta_{\alpha 1}) \right) \right. \\ &\quad \left. - \frac{ax}{E} |U_{\alpha 3}|^2 |U_{13}|^2 \sin 2\Delta_{31} \right] \\ &\quad + 4 \sin^2 \Delta_{31} |U_{\beta 3}|^2 |U_{\alpha 3}|^2 \left[1 - 2 \frac{a}{\Delta m_{31}^2} (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}) \right] \\ &\quad - 8 \Delta_{21} \sin^2 \Delta_{31} \operatorname{Im}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) \\ &\quad + 4 \sin 2\Delta_{31} \left[\Delta_{21} \operatorname{Re}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) \right. \\ &\quad \left. + \frac{ax}{4E} (|U_{13}|^2 \delta_{\alpha 1} \delta_{\beta 1} + |U_{\beta 3}|^2 |U_{\alpha 3}|^2 (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1})) \right] \\ &\quad + 4 \Delta_{21}^2 |U_{\beta 2}|^2 |U_{\alpha 2}|^2 \end{aligned} \quad (2)$$

Survival probability $P(\nu_\mu \rightarrow \nu_\mu)$ in matter

For $\alpha = \beta = \mu$ we have

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_\mu) = & 1 \\ & + 4 \sin^2 \Delta_{31} |U_{\mu 3}|^2 \left[(|U_{\mu 3}|^2 - 1) - \frac{2a}{\Delta m_{31}^2} |U_{e 3}|^2 (2|U_{\mu 3}|^2 - 1) \right] \\ & + 4 \Delta_{31} \sin 2\Delta_{31} |U_{\mu 3}|^2 \left[\frac{\Delta m_{21}^2}{\Delta m_{31}^2} |U_{\mu 2}|^2 + \frac{a}{\Delta m_{31}^2} |U_{e 3}|^2 (2|U_{\mu 3}|^2 - 1) \right] \\ & + 4 \Delta_{21}^2 |U_{\mu 2}|^4 \end{aligned} \quad (3)$$

Survival probability $P(\nu_\mu \rightarrow \nu_\mu)$ in matter

In term of PMNS mixing angles, we see that

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_\mu) &= 1 + 4s_{23}^2 c_{13}^2 (s_{23}^2 c_{13}^2 - 1) \sin^2 \Delta_{31} \\ &+ 4s_{23}^2 c_{13}^2 s_{13}^2 (2s_{23}^2 c_{13}^2 - 1) \frac{2a}{\Delta m_{31}^2} \sin^2 \Delta_{31} \\ &+ 4s_{23}^2 c_{13}^2 s_{13}^2 (2s_{23}^2 c_{13}^2 - 1) \frac{a}{\Delta m_{31}^2} \Delta_{31} \sin 2\Delta_{31} \\ &+ 4s_{23}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta) \Delta_{21} \sin 2\Delta_{31} \\ &+ 4(c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta)^2 \Delta_{21}^2 \end{aligned} \quad (4)$$

Transition probability $P(\nu_\mu \rightarrow \nu_e)$ in matter

For $\alpha = \mu$ and $\beta = e$ we have

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) = & 4 \sin^2 \Delta_{31} |U_{e3}|^2 |U_{\mu 3}|^2 \\ & - 8 \sin^2 \Delta_{31} |U_{e3}|^2 |U_{\mu 3}|^2 \frac{a}{\Delta m_{31}^2} (2|U_{e3}|^2 - 1) \\ & + 4 \sin 2\Delta_{31} \frac{ax}{4E} |U_{e3}|^2 |U_{\mu 3}|^2 (2|U_{e3}|^2 - 1) \\ & - 8\Delta_{21} \sin^2 \Delta_{31} \text{Im}(U_{e3}^* U_{\mu 3} U_{e2} U_{\mu 2}^*) \\ & + 4\Delta_{21} \sin 2\Delta_{31} \text{Re}(U_{e3}^* U_{\mu 3} U_{e2} U_{\mu 2}^*) \\ & + 4\Delta_{21}^2 |U_{e2}|^2 |U_{\mu 2}|^2 \end{aligned} \quad (5)$$

Transition probability $P(\nu_\mu \rightarrow \nu_e)$ in matter

Insert the PMNS matrix elements, we have

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) = & 4s_{13}^2 s_{23}^2 c_{13}^2 \sin^2 \Delta_{31} \\ & - 8s_{13}^2 s_{23}^2 c_{13}^2 \frac{a}{\Delta m_{31}^2} (2s_{13}^2 - 1) \sin^2 \Delta_{31} \\ & + 4s_{13}^2 s_{23}^2 c_{13}^2 \frac{ax}{4E} (2s_{13}^2 - 1) \sin 2\Delta_{31} \\ & - 8s_{12}s_{13}s_{23}c_{12}c_{13}^2c_{23} \sin \delta \Delta_{21} \sin^2 \Delta_{31} \\ & + 4s_{12}s_{13}s_{23}c_{13}^2 (c_{12}c_{23} \cos \delta - s_{12}s_{13}s_{23}) \Delta_{21} \sin 2\Delta_{31} \\ & + 4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12}s_{13}s_{23}c_{12}c_{23} \cos \delta) \Delta_{21}^2 \quad (6) \end{aligned}$$

Transition probability $P(\nu_\mu \rightarrow \nu_e)$ in matter

For $\frac{\Delta m_{21}^2 x}{4E} \ll 1$ and $\Delta m_{31}^2 \approx \Delta m_{32}^2$, by making a replacement with: $\Delta_{21} = \sin \Delta_{21}$; $\cos \Delta_{31} = \cos \Delta_{32}$; $\sin \Delta_{31} = \sin \Delta_{32}$ and we get

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) = & 4s_{13}^2 s_{23}^2 c_{13}^2 \sin^2 \Delta_{31} & (7) \\ & - 8s_{13}^2 s_{23}^2 c_{13}^2 \frac{a}{\Delta m_{31}^2} (2s_{13}^2 - 1) \sin^2 \Delta_{31} \\ & + 8s_{13}^2 s_{23}^2 c_{13}^2 \frac{ax}{4E} (2s_{13}^2 - 1) \sin \Delta_{31} \cos \Delta_{32} \\ & - 8s_{12} s_{13} s_{23} c_{12} c_{13}^2 c_{23} \sin \delta \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\ & + 8s_{12} s_{13} s_{23} c_{13}^2 (c_{12} c_{23} \cos \delta - s_{12} s_{13} s_{23}) \sin \Delta_{21} \sin \Delta_{31} \cos \Delta_{32} \\ & + 4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta) \sin^2 \Delta_{21} \end{aligned}$$

Transition probability $P(\nu_\mu \rightarrow \nu_e)$ in matter

We can see from the equation (7) that

- The first term dominates
- The second and the third terms involve matter effect
- The fourth term is CP-violating term
- The fifth term is CP-conserving term
- The last one is solar neutrino

Next to do

- Get familiar with Ubuntu and using ROOT in Ubuntu
- Make plots of oscillation probability in matter
- Learn Statistics
- Learn about T2K experiment

References

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